Philosophical Psychology
Vol. 22, No. 1, February 2009, 75–79

Does topological perception rest on a misconception about topology?

Roberto Casati

In this article I assess some results that purport to show the existence of a type of 'topological perception', i.e., perceptually based classification of topological features. Striking findings about perception in insects appear to imply that (1) configural, global properties can be considered as primitive perceptual features, and (2) topological features in particular are interesting as they are amenable to formal treatment. I discuss four interrelated questions that bear on any interpretation of findings about the perception of topological properties: what exactly are topological properties, what makes them global, in what sense the quoted findings makes them primitive, and what are the hopes of a formal theory of perception based upon them. I suggest that mathematical topology is not the correct model for cognition topological properties, hence that some other formalism ought to be used—a form of ''internalized topology.'' However, once the principles of this type of topology are spelled out, they may not be as globalistic as one may have expected.

Keywords: Topological Perception; Topology; Visual Primitives

Many logically independent but coordinated factors constrain the quest for visual primitives. First, phenomenology tells us that the visual scene is complex, but at the same time that there are recurring elements out of which complexity may be built (Kanizsa, 1979). Second, mathematical models show how it is possible to build complex representations out of representations of simpler components (Biederman, 1987). Third, computational architecture makes it plausible that the complexity of retinal input (providing pixel size information about properties at places) be

Roberto Casati is 22 at 22.
Correspondence to: Roberto Casati, 22. Email: casati@ehess.fr

ISSN 0951-5089 (print)/ISSN 1465-394X (online)/09/010075-5 © 2009 Taylor & Francis
DOI: 10.1080/09515080802703711

organized at very early stages into elemental descriptors to reduce the computational load of subsequent stages (Palmer & Rock, 1994). Finally, behavioral and neurophysiological evidence for specific, down to single-neuron sensitivity to relatively well delineated features of the environment has been gathered over the last decades, starting from (Hubel & Wiesel, 1959). But do these criteria converge on a single list of primitives? They do not have to, of course; and finding out that what we expect to be phenomenologically or computationally primitive is not so behaviorally or neurophysiologically will make for an interesting discovery. This is in part the interest of (Chen, Zhang, & Srinivasan, 2003) finding that small brains such as those of the honey bees display a sensitivity to global configurational properties, in particular topological properties such as the presence or the absence of holes in 2-d displays. It looks as if not only bees are able to distinguish between configurations that differ only in their topological properties, but also they are able to generalize to topologically equivalent configurations that are rather different on many other respects. According to (Pomerantz, 2003), the findings are interesting for two reasons. The first reason is that the topological properties in question are generally considered as relatively complex and hard to compute (Minsky & Papert, 1998) but at the same time are very deep and robust properties of the environment—they are invariant under most transformations, as opposed, say, to metric properties, hence sensitivity to them would have a high adaptive value. The second reason is that the mathematics behind topological features is sufficiently well understood and formalized accordingly, as opposed to the relatively informality of characterizations of global features, e.g., the one found in the gestalt literature.

But what exactly are topological properties, what makes them global, in what sense Chen et al.'s (2003) finding makes them primitive, and what are the hopes of a formal theory of perception based upon them?

Let us first briefly review the results Chen et al.'s experiment. Honeybees were trained to choose one among a pair of configurations: an O-shaped stimulus and an S-shaped stimulus, say. Then they were tested on their ability to distinguish the O-shaped stimulus from other stimuli that are either topologically nonequivalent to the O-shape (such as a Â-shape, or a f -shape) and stimuli that are topologically equivalent to the O-shape (such as a -shape) but look different as to their non-topological aspect. Bees succeeded in making the distinction with the first set of stimuli but failed to make it with the second set. This indicates both that they were sensitive to topological differences and that they correctly lumped together items that are topologically equivalent.

One should note in the first place that the displays used by Chen et al. for testing honeybees were 2-d pictures representing figures of different shapes and varying topological properties. There is, of course, a much general problem of using 2-d stimuli in order to draw inferences about a visual system that has adapted to a 3-d world. But there is also a specific problem: topology in 3-d is not automatically mapped onto 2-d topology. A 2-d image like the shape of the letter B can be the projection of a 3-d letter O that has bent over in the middle. Hence from sensitivity to 2-d topology one can only infer with much care to a corresponding sensitivity to the topology of 3-d bodies; this by itself would question any ecological-adaptive considerations.

The globality of topological properties can be captured intuitively in opposition to the locality of other features. The directionality of a line, for instance, is an intrinsically local matter. At point p the line has a direction that is given by its tangent at p. Little does it matter how the line looks like at a (sufficient) distance from p. On the other hand, the fact that the line closes unto itself (like a circle) or has terminations (like a bar) cannot be made depend on the properties of a single point on the line; many other points have to be scrutinized. In this sense the properties studied by Chen et al. are global, and appear to be the subject matter of topology.

The main question arises whether the term 'topology' is used in the loose and popular sense of 'rubber sheet geometry' or by reference to formal mathematical notions. The issue of control in experimental design reflects this uncertainty. Chen et al. appropriately point out that it is hard to test for topological differences without introducing some non-topological differences in the stimuli: ''there seem to be, in principle, no two geometric features that differ only in topological properties'' (2003, p. 6687). But from the viewpoint of mathematical topology, this is inaccurate. An open sphere and a closed sphere—or their 2-d equivalents, a closed circle and an open circle—have different topological properties but the same metric properties (same radius, as the boundary of the closed circle has no dimension). To be sure, this fact may not have any consequence for the design of visual test stimuli, as the difference between a closed and an open item has no visual counterpart (dimensionless items, one may argue, are under perceptual discrimination threshold). But this brings us to an important point. When we talk about topological differences in visual displays, we may not be talking about the differences that are the subject matter of mathematical topology. Hence talking of 'topology' requires some other refinement, short of being loose talk, especially if it is to provide the 'formal theory' that (Pomerantz, 2003) invokes.

One may suggest that something like an internalized topology captures the features we have in mind—such as the ability to sort out objects based on the number of holes they have (the difference between the letters B and O, having one and two holes respectively), or to assess the equivalence between figures (say, the letter S and the letter I). But here we have to exert some care, because the theory now requires a new notion, such as hole, and some account is needed of what it is for the visual system to process the feature of being a hole in such a way that it contributes to the explanation of the performance of distinguishing S or B from O.

To see the point more clearly, consider a re-interpretation of Chen et al. (2003). From the viewpoint of an intuitive topology, the difference between having and not having one hole is implied by the presence or absence of other visual features. Now, if it were demonstrated that processing of these features is available to the visual system, it would be possible to reassess the claim that the global features invoked by Chen et al. and Pomerantz (2003) are perceptual primitives. The following is a proposal in that sense.

The visual features in question are:

(1) The presence of complete visual boundaries of a unit, and
(2) The uniformity and connection of the unit (Palmer & Rock, 1994), along with its maximality (Casati, 2002).

The presence of holes is correlated with these simpler features in the following way. If a maximal uniform connected unit possesses just one complete visual boundary, then it has no hole. If it possesses two visual boundaries, then it has one hole. In general,[1] for any given visual display:

(3) For m maximal uniform connected figures and n complete visual boundaries, the number of holes is (n–m).

The further element that is then needed is that the visual system implement some way of counting the features and compare their cardinalities. Given what is known about the limits of the ability to subitize small quantities, it is expected that the difference between configurations with, say, one and two holes will be accessible to the system. At the same time, the difference between configurations with nine and ten holes is expected not be accessible to the system. But surely these latter configurations are topologically distinct from the viewpoint of mathematical topology. Hence testing the ability of distinguishing between configurations with varying numbers of holes can decide between a holistic and a less holistic account of visual properties.

Furthermore, how far can topological generalization go? Letters I and J are topologically equivalent in the intended sense; but so are, presumably, I, L, K and H (the latter three, for instance, can all be ‘shrunk’ to an I without ‘cutting or gluing’). Will data confirm a sensitivity to these equivalences? Some may expect instead that some sort of parsing by components will predict that these shapes are resilient to placement in a single category: an H has three components, an I has only one. Here again the globalistic hypothesis can be pitted against other theoretical accounts.

It may be questioned whether features (1) and (2) are really simpler than the global feature of having a hole. After all, both (1) and (2) presuppose that the unity (connection) of both the boundary and the figure are accessed, and assessing connection is a notoriously difficult computational problem. However, on the one hand, this is a general problem, one that affects all theories that are supposed to characterize the entry units of the visual system. On the other hand, in order to show that sensitivity to the feature of possessing a hole is not sensitivity to a visual primitive, it is enough to show that the former can be explained in terms of sensitivity to other features, without any further commitment to the hypothesis that these features are themselves visual primitives.

To conclude, what is the evidence that topological or global features such as having a hole are primitives of the system, according to Chen et al. (2003)? The clearly delineated criterion in the paper is the size of the computational system: honeybees have small brains. The criterion is novel relative to the four criteria listed at the top of this paper. The criterion predicts that a feature is primitive if it is computed by a small system. However, for the reasons given above, the system in question may simply be not small enough to provide a cogent answer.

Note

[1] The criterion reflects the one given for cavities in 3-d bodies in (Casati & Varzi, 1994).

References

Biederman, I. (1987). Recognition-by-components: A theory of human image interpretation. Psychological Review, 94, 115–148.

Casati, R. (2002). Topology and cognition. Encyclopedia of cognitive science (pp. 2–2). 2: McMillan.

Casati, R., & Varzi, A. C. (1994). Holes and other superficialities. Cambridge, MA: MIT Press.

Chen, L., Zhang, S., & Srinivasan, M. V. (2003). Global perception in small brains: Topological pattern recognition in honeybees. PNAS, 100(11), 6884–6889.

Hubel, D. H., & Wiesel, T. N. (1959). Receptive fields of single neurons in cat's striate cortex. Journal of Physiology, 160, 106–154.

Kanizsa, G. (1979). Organization in vision: Essays on gestalt perception. 2: Praeger.

Minsky, M. L., & Papert, S. A. (1998). Perceptrons: An introduction to computational geometry (expanded edition). Cambridge, MA: MIT Press.

Palmer, S., & Rock, I. (1994). Rethinking perceptual organization: The role of uniform connectedness. Psychonomic Bulletin and Review, 1, 29–55.

Pomerantz, J. R. (2003). Wholes, holes and basic features in vision. Trends in Cognitive Sciences, 7(11), 471–473.